Power

Power — the Work Done Every Second

In mechanical situations, **whenever** energy is **converted**, **work** is being done.
For example, when an object is **falling**, the force of **gravity** is doing work on that object **equal** to the **increase** in **kinetic energy** (ignoring air resistance).
The **rate** at which this work is being done is called the **power**.
You can calculate it using:

power (in watts) = **work done** (in joules) ÷ **time taken** (in seconds)

Or, in symbols: $P = \frac{W}{t}$

Power is measured in **watts**.
A watt is equivalent to **one joule of work done per second.**

EXAMPLE: If 10 joules of work are done in 2 seconds, what is the power?

$P = W \div t = 10 \div 2 = \mathbf{5\ W}$

EXAMPLE: For how long must a 3.2 kilowatt (3.2×10^3 watt) engine run to do 480 kilojoules (4.8×10^5 joules) of work?

$P = W \div t$

Multiplying both sides by t gives: $P \times t = W$

Then dividing both sides by P gives: $t = W \div P$

So, $t = W \div P = \frac{4.8 \times 10^5}{3.2 \times 10^3} = \mathbf{150\ s}$

EXAMPLE: A force of 125 newtons pushes a crate 5.2 metres in 2.6 seconds. What is the power? (The motion is in the same direction as the force.)

First you need to find the work done (see page 18):

$W = F \times s = 125 \times 5.2 = 650$ J

Then use W to find the power:

$P = W \div t = 650 \div 2.6 = \mathbf{250\ W}$

The power of love ain't that special — it's just a lot of work over time...

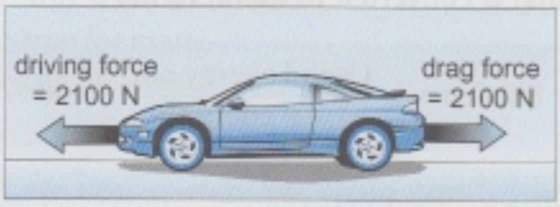
1) What is the power output of a motor if it does 250 joules of work in 4.0 seconds?
2) If a lift mechanism works at 14 kilowatts, how long does it take to do 91 kilojoules of work?
3) An engine provides a force of 276 N to push an object 1.25 km in 2.5 minutes. What power is the engine working at?

Power

Power is also **Force** Multiplied By **Speed**

There's a **useful equation** you can **derive** for the **work done** by a force **every second** on an object moving at a **constant speed**. Follow through the working in the example below:

EXAMPLE: What power is a car engine working at if it produces a driving force of 2100 newtons when moving at a steady speed of 32 metres per second?



The car is moving at a steady speed. This means the forces on it are balanced, so the driving force must be equal to the drag force.

The power of the engine is given by $P = W \div t$.

$W = F \times s$, so we can substitute for the work done, giving $P = \frac{F \times s}{t}$.

Now, $\frac{F \times s}{t}$ is the same as $F \times \frac{s}{t}$, so $P = F \times \frac{s}{t}$.

Finally we use the fact that $\frac{s}{t} = \frac{\text{distance travelled}}{\text{time taken}}$ = the speed, v.

So, $P = F \times \frac{s}{t} = F \times v$

power (in watts) = **force** (in newtons) × **speed** (in metres per second)

For our example, $P = 2100 \times 32 = 67\,200 =$ **67 000 W** (or 67 kW) **(to 2 s.f.)**

(This answer is rounded to 2 s.f. to match the data in the question — see page 1.)

<u>IMPORTANT</u>:
The formula $P = F \times v$ is **only** true when the object is moving at a **constant speed** in the **same direction as the force**.

Mooving forces with a lot of power — a stampeding herd of cows...

1) What is the power delivered by a train engine if its driving force of 1.80×10^5 newtons produces a constant speed of 40.0 metres per second?
2) A skydiver is falling at a constant velocity of 45 metres per second. Gravity is doing work on her at a rate of 31 500 joules per second. What is her weight?
3) A car is travelling at steady speed. Its engine delivers a power of 5.20×10^4 watts to provide a force of 1650 newtons. What speed is the car travelling at (in metres per second)?

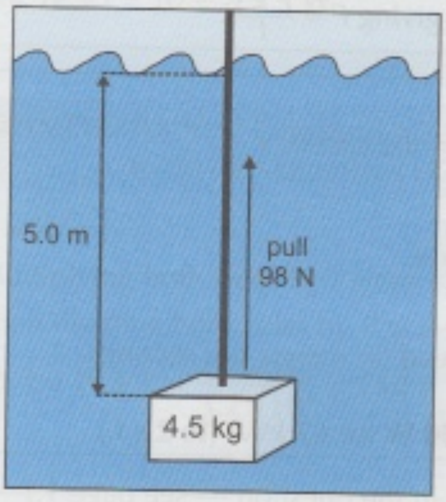
Efficiency

How Much of What You **Put In** Do You **Get Out**?

1) For most mechanical systems you **put in** energy in **one form** and the system **gives out** energy in **another**.
2) However, **some** energy is **always** converted into forms that **aren't useful**.
3) For example, an electric motor converts electrical energy into **heat** and **sound** as well as useful kinetic energy.
4) You can measure the **efficiency** of a system by the **percentage of total energy put in that is converted to useful forms.**

$$\text{Efficiency} = \frac{\text{Useful energy out}}{\text{Total energy in}} \times 100\%$$

EXAMPLE: A pirate uses a rope to pull a box of mass 4.5 kg vertically upwards through 5.0 m of water. He pulls with a force of 98 N. What is the efficiency of this system?



The **energy the pirate puts in** is the work he does pulling the rope.
The **useful energy out** is the gravitational potential energy gained by the box.
Some energy is converted to heat and sound by **friction** as the box is dragged through the water.

Total energy in = work done = $F \times s$
= 98×5.0
= 490 J

Useful energy out = gravitational potential energy gained
= $m \times g \times h$
= $4.5 \times 9.81 \times 5.0$
= 220.725 J

So, efficiency = $\frac{\text{Useful energy out}}{\text{Total energy in}} \times 100\%$

$= \frac{220.725}{490} \times 100\% = 45.045... =$ **45% (to 2 s.f.)**

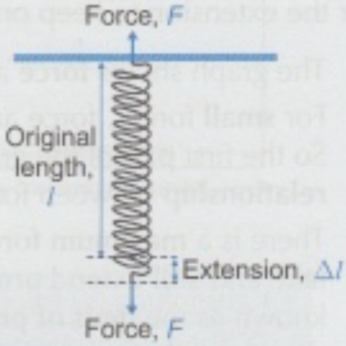
Efficiency – getting on with these questions instead of messing about...

1) A motor uses 375 joules of electrical energy in lifting a 12.9 kilogram mass through 2.50 metres. What is its efficiency?
2) It takes 1.4 megajoules (1.4×10^6 joules) of chemical energy from the petrol in a car engine to accelerate a 560 kilogram car from rest to 25 metres per second on a flat road.
   a) What is the gain in kinetic energy?
   b) What is the efficiency of the car?

Forces and Springs

Hooke's Law — *Extension* is *Directly Proportional* to *Force*

1) When you apply a **force** to an object you can cause it to **stretch** and **deform** (change shape).
2) **Elastic objects** are objects that return to their **original shape** after this deforming force is **removed**, e.g. springs.
3) When a **spring** is supported at the top and a **weight** is attached to the bottom, it **stretches**.
4) The **extension**, Δl, of a spring is **directly proportional** to the **force** applied, F. This is **Hooke's Law**.
5) This relationship is also true for many other elastic objects like **metal wires**.



force (in newtons, N) = **spring constant** (in newtons per metre, Nm^{-1}) × **extension** (in metres, m)

$$F = k \times \Delta l$$

The **spring constant**, k, depends on the stiffness of the **material** that you are stretching. It's measured in **newtons per metre** (Nm^{-1}).

EXAMPLE: A force is applied to a spring with a spring constant of 65.0 Nm^{-1}. The spring extends by 12.3 cm. What size is the force?

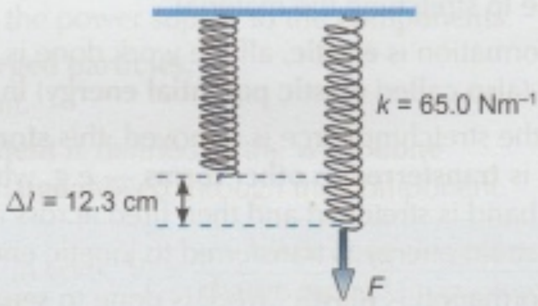
$F = k \times \Delta l$

$\Delta l = 12.3$ cm $= 0.123$ m

So, $F = 65.0 \times 0.123$

$= 7.995$

$=$ **8.00 N (to 3 s.f.)**



EXAMPLE: A sack of flour of mass 7.10 kg is attached to the bottom of a vertical spring. The spring constant is 85.0 Nm^{-1} and the top of the spring is supported. How much does the spring extend by?

$F = k \times \Delta l$, so $\Delta l = \frac{F}{k}$

You need to work out the force from the given mass:

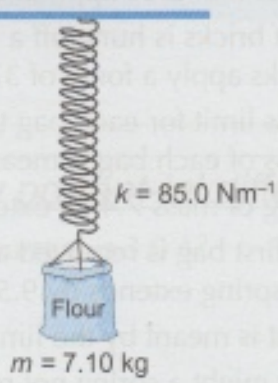
$F =$ weight of flour $= m \times g$

$= 7.10 \times 9.81 = 69.651$ N

So, $\Delta l = \frac{69.651}{85.0}$

$= 0.8194...$

$=$ **0.819 m (to 3 s.f.)**

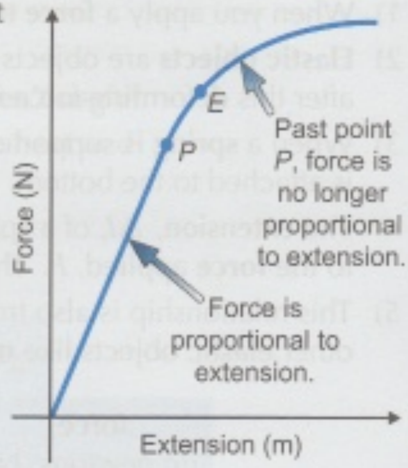


Forces and Springs

Hooke's Law **Stops Working** when the **Force** is **Great Enough**

There's a **limit** to the amount of force you can apply to an object for the extension to keep on increasing **proportionally**.

1) The graph shows **force** against **extension** for a spring.
2) For **small** forces, force and extension are **proportional**. So the first part of the graph shows a **straight-line relationship** between force and extension.
3) There is a **maximum force** that the spring can take and **still extend proportionally**. This is known as the **limit of proportionality** and is shown on the graph at the point marked ***P***.
4) The point marked ***E*** is the **elastic limit**. If you increase the force past this point, the spring will be **permanently stretched**. When the force is **removed**, the spring will be **longer** than at the start.
5) Beyond the **elastic limit**, we say that the spring deforms **plastically**.



Work Done can be **Stored** as **Elastic Strain Energy**

1) When a material is **stretched**, **work** has to be done in stretching the material.
2) If a deformation is **elastic**, all the work done is **stored** as **elastic strain energy** (also called **elastic potential energy**) in the material.
3) When the stretching force is removed, this **stored energy** is **transferred** to **other forms** — e.g. when an elastic band is stretched and then fired across a room, elastic strain energy is transferred to kinetic energy.
4) If a deformation is **plastic**, work is done to **separate atoms**, and energy is **not** stored as strain energy (it's mostly lost as heat).

Spring into action — force yourself to learn all this...

1) A force applied to a spring with spring constant 64.1 Nm^{-1} causes it to extend by 24.5 cm. What was the force applied to the spring?
2) A pile of bricks is hung off a spring with spring constant 84.0 Nm^{-1}. The bricks apply a force of 378 N on the spring. How much does the spring extend by?
3) The mass limit for each bag taken on a flight with Cheapskate Airways is 9.0 kg. The mass of each bag is measured by attaching the bag to a spring.
   a) A bag of mass 7.4 kg extends the spring by 8.4 cm. What is the spring constant?
   b) The first bag is removed and another bag is attached to the spring. The spring extends by 9.5 cm. Can this bag be taken on the flight?
4) a) What is meant by the limit of proportionality?
   b) Why might a spring not return to its original length after having been stretched and then released?