Power

Power — the Work Done Every Second

In mechanical situations, **whenever** energy is **converted**, **work** is being done.
For example, when an object is **falling**, the force of **gravity** is doing work on that object **equal** to the **increase** in **kinetic energy** (ignoring air resistance).
The **rate** at which this work is being done is called the **power**.
You can calculate it using:

power (in watts) = **work done** (in joules) ÷ **time taken** (in seconds)

Or, in symbols: $P = \frac{W}{t}$

Power is measured in **watts**.
A watt is equivalent to **one joule of work done per second**.

EXAMPLE: If 10 joules of work are done in 2 seconds, what is the power?

$P = W \div t = 10 \div 2 = \mathbf{5\ W}$

EXAMPLE: For how long must a 3.2 kilowatt (3.2×10^3 watt) engine run to do 480 kilojoules (4.8×10^5 joules) of work?

$P = W \div t$
Multiplying both sides by t gives: $P \times t = W$
Then dividing both sides by P gives: $t = W \div P$
So, $t = W \div P = \frac{4.8 \times 10^5}{3.2 \times 10^3} = \mathbf{150\ s}$

EXAMPLE: A force of 125 newtons pushes a crate 5.2 metres in 2.6 seconds. What is the power? (The motion is in the same direction as the force.)

First you need to find the work done (see page 18):
$W = F \times s = 125 \times 5.2 = 650$ J
Then use W to find the power:
$P = W \div t = 650 \div 2.6 = \mathbf{250\ W}$

The power of love ain't that special — it's just a lot of work over time...

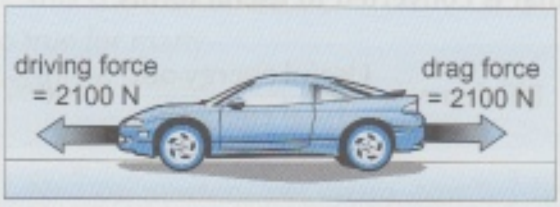
1) What is the power output of a motor if it does 250 joules of work in 4.0 seconds?
2) If a lift mechanism works at 14 kilowatts, how long does it take to do 91 kilojoules of work?
3) An engine provides a force of 276 N to push an object 1.25 km in 2.5 minutes. What power is the engine working at?

Power

Power is also Force Multiplied By Speed

There's a **useful equation** you can **derive** for the **work done** by a force **every second** on an object moving at a **constant speed**. Follow through the working in the example below:

EXAMPLE: What power is a car engine working at if it produces a driving force of 2100 newtons when moving at a steady speed of 32 metres per second?



The car is moving at a steady speed. This means the forces on it are balanced, so the driving force must be equal to the drag force.

The power of the engine is given by $P = W \div t$.

$W = F \times s$, so we can substitute for the work done, giving $P = \frac{F \times s}{t}$.

Now, $\frac{F \times s}{t}$ is the same as $F \times \frac{s}{t}$, so $P = F \times \frac{s}{t}$.

Finally we use the fact that $\frac{s}{t} = \frac{\text{distance travelled}}{\text{time taken}}$ = the speed, v.

So, $P = F \times \frac{s}{t} = F \times v$

power (in watts) = **force** (in newtons) × **speed** (in metres per second)

For our example, $P = 2100 \times 32 = 67\,200 =$ **67 000 W** (or 67 kW) **(to 2 s.f.)**

(This answer is rounded to 2 s.f. to match the data in the question — see page 1.)

IMPORTANT:
The formula $P = F \times v$ is **only** true when the object is moving at a **constant speed** in the **same direction as the force**.

Mooving forces with a lot of power — a stampeding herd of cows...

1) What is the power delivered by a train engine if its driving force of 1.80×10^5 newtons produces a constant speed of 40.0 metres per second?
2) A skydiver is falling at a constant velocity of 45 metres per second. Gravity is doing work on her at a rate of 31 500 joules per second. What is her weight?
3) A car is travelling at steady speed. Its engine delivers a power of 5.20×10^4 watts to provide a force of 1650 newtons. What speed is the car travelling at (in metres per second)?

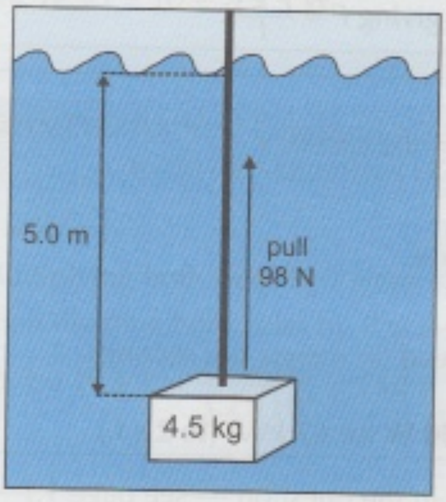
Efficiency

How Much of What You **Put In** Do You **Get Out**?

1) For most mechanical systems you **put in** energy in **one form** and the system **gives out** energy in **another**.
2) However, **some** energy is **always** converted into forms that **aren't useful**.
3) For example, an electric motor converts electrical energy into **heat** and **sound** as well as useful kinetic energy.
4) You can measure the **efficiency** of a system by the **percentage of total energy put in that is converted to useful forms.**

$$\text{Efficiency} = \frac{\text{Useful energy out}}{\text{Total energy in}} \times 100\%$$

EXAMPLE: A pirate uses a rope to pull a box of mass 4.5 kg vertically upwards through 5.0 m of water. He pulls with a force of 98 N. What is the efficiency of this system?



The **energy the pirate puts in** is the work he does pulling the rope.
The **useful energy out** is the gravitational potential energy gained by the box.
Some energy is converted to heat and sound by **friction** as the box is dragged through the water.

Total energy in = work done = $F \times s$
= 98 × 5.0
= 490 J

Useful energy out = gravitational potential energy gained
= $m \times g \times h$
= 4.5 × 9.81 × 5.0
= 220.725 J

So, efficiency = $\frac{\text{Useful energy out}}{\text{Total energy in}} \times 100\%$

$= \frac{220.725}{490} \times 100\% = 45.045... =$ **45% (to 2 s.f.)**

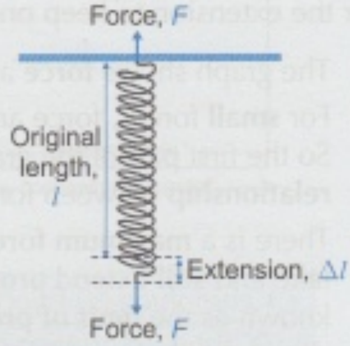
Efficiency – getting on with these questions instead of messing about...

1) A motor uses 375 joules of electrical energy in lifting a 12.9 kilogram mass through 2.50 metres. What is its efficiency?
2) It takes 1.4 megajoules (1.4×10^6 joules) of chemical energy from the petrol in a car engine to accelerate a 560 kilogram car from rest to 25 metres per second on a flat road.
   a) What is the gain in kinetic energy?
   b) What is the efficiency of the car?

Forces and Springs

Hooke's Law — Extension is Directly Proportional to Force

1) When you apply a **force** to an object you can cause it to **stretch** and **deform** (change shape).
2) **Elastic objects** are objects that return to their **original shape** after this deforming force is **removed**, e.g. springs.
3) When a **spring** is supported at the top and a **weight** is attached to the bottom, it **stretches**.
4) The **extension**, Δl, of a spring is **directly proportional** to the **force** applied, F. This is **Hooke's Law**.
5) This relationship is also true for many other elastic objects like **metal wires**.



force (in newtons, N) = **spring constant** (in newtons per metre, Nm^{-1}) × **extension** (in metres, m)

$$F = k \times \Delta l$$

The **spring constant**, k, depends on the stiffness of the **material** that you are stretching. It's measured in **newtons per metre** (Nm^{-1}).

EXAMPLE: A force is applied to a spring with a spring constant of 65.0 Nm^{-1}. The spring extends by 12.3 cm. What size is the force?

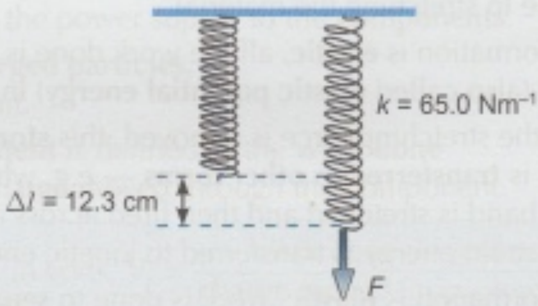
$F = k \times \Delta l$

$\Delta l = 12.3$ cm $= 0.123$ m

So, $F = 65.0 \times 0.123$

$= 7.995$

$=$ **8.00 N (to 3 s.f.)**



EXAMPLE: A sack of flour of mass 7.10 kg is attached to the bottom of a vertical spring. The spring constant is 85.0 Nm^{-1} and the top of the spring is supported. How much does the spring extend by?

$F = k \times \Delta l$, so $\Delta l = \frac{F}{k}$

You need to work out the force from the given mass:

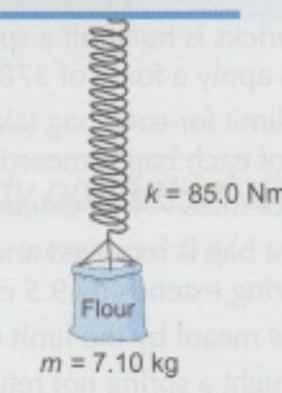
$F =$ weight of flour $= m \times g$

$= 7.10 \times 9.81 = 69.651$ N

So, $\Delta l = \frac{69.651}{85.0}$

$= 0.8194...$

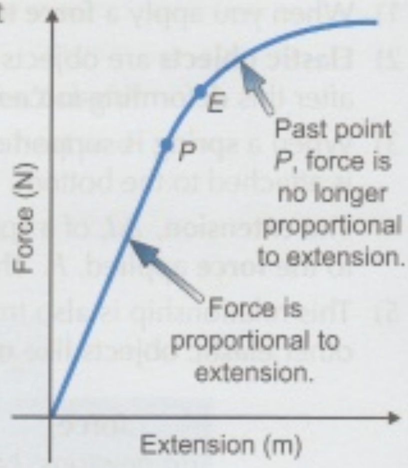
$=$ **0.819 m (to 3 s.f.)**



Forces and Springs

*Hooke's Law **Stops Working** when the **Force** is **Great Enough***

There's a **limit** to the amount of force you can apply to an object for the extension to keep on increasing **proportionally**.



1) The graph shows **force** against **extension** for a spring.
2) For **small** forces, force and extension are **proportional**. So the first part of the graph shows a **straight-line relationship** between force and extension.
3) There is a **maximum force** that the spring can take and **still extend proportionally**. This is known as the **limit of proportionality** and is shown on the graph at the point marked ***P***.
4) The point marked ***E*** is the **elastic limit**. If you increase the force past this point, the spring will be **permanently stretched**. When the force is **removed**, the spring will be **longer** than at the start.
5) Beyond the **elastic limit**, we say that the spring deforms **plastically**.

*Work Done can be **Stored** as **Elastic Strain Energy***

1) When a material is **stretched**, **work** has to be done in stretching the material.
2) If a deformation is **elastic**, all the work done is **stored** as **elastic strain energy** (also called **elastic potential energy**) in the material.
3) When the stretching force is removed, this **stored energy** is **transferred** to **other forms** — e.g. when an elastic band is stretched and then fired across a room, elastic strain energy is transferred to kinetic energy.
4) If a deformation is **plastic**, work is done to **separate atoms**, and energy is **not** stored as strain energy (it's mostly lost as heat).

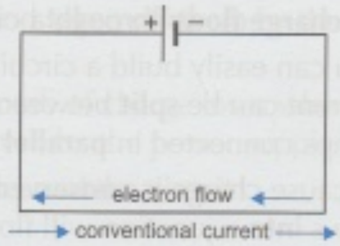
Spring into action — force yourself to learn all this...

1) A force applied to a spring with spring constant 64.1 Nm^{-1} causes it to extend by 24.5 cm. What was the force applied to the spring?
2) A pile of bricks is hung off a spring with spring constant 84.0 Nm^{-1}. The bricks apply a force of 378 N on the spring. How much does the spring extend by?
3) The mass limit for each bag taken on a flight with Cheapskate Airways is 9.0 kg. The mass of each bag is measured by attaching the bag to a spring.
   a) A bag of mass 7.4 kg extends the spring by 8.4 cm. What is the spring constant?
   b) The first bag is removed and another bag is attached to the spring. The spring extends by 9.5 cm. Can this bag be taken on the flight?
4) a) What is meant by the limit of proportionality?
   b) Why might a spring not return to its original length after having been stretched and then released?

Current and Potential Difference

Electric Current — the Rate of Flow of Charge Around a Circuit

1) In a circuit, **negatively-charged electrons** flow from the **negative** end of a battery to the **positive** end.
2) This flow of charge is called an **electric current.**
3) However, you can also think of current as a flow of **positive charge** in the **other direction**, from **positive** to **negative**. This is called **conventional current**.



The electric current at a point in the wire is defined as:

$$\textbf{current} \text{ (in amperes, A)} = \frac{\textbf{the amount of charge} \text{ passing the point (in coulombs, C)}}{\textbf{the time it takes} \text{ for the charge to pass (in seconds, s)}}$$

Or, in symbols: $I = \frac{Q}{t}$

EXAMPLE: 585 C of charge passes a point in a circuit in 45.0 s. What is the current at this point?

$I = \frac{Q}{t}$, so $I = \frac{585}{45.0}$ = **13.0 A**

Potential Difference (Voltage) — the Energy Per Unit Charge

1) In all circuits, energy is **transferred** from the power supply to the **components**.
2) The **power supply** does **work** on the **charged particles**, which **carry** this energy **around** the circuit.
3) The potential difference **across a component** is defined as the **work done** (or energy transferred) **per coulomb** of charge moved through the component.

$$\textbf{Potential difference across component} \text{ (in volts, V)} = \frac{\textbf{work done} \text{ (in joules, J)}}{\textbf{charge moved} \text{ (in coulombs, C)}}$$

In symbols: $V = \frac{W}{Q}$

EXAMPLE: A component does 10.8 J of work for every 2.70 C that passes through it. What is the potential difference across the component?

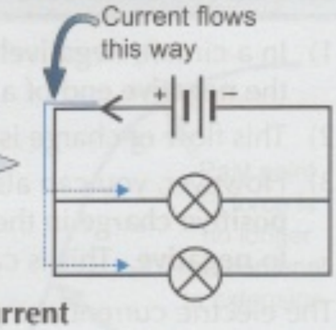
$V = \frac{W}{Q}$, so $V = \frac{10.8}{2.70}$ = **4.00 V**

Physicists love camping trips — they get to study po-tent-ial difference...

1) How long does it take to transfer 12 C of charge if the average current is 3.0 A?
2) The potential difference across a bulb is 1.5 V. How much work is done to pass 9.2 C through the bulb?
3) A motor runs for 275 seconds and does 9540 J of work. If the current in the circuit is 3.80 A, what is the potential difference across the motor?

Current in Electric Circuits

*Charge is **Always Conserved** in Circuits*

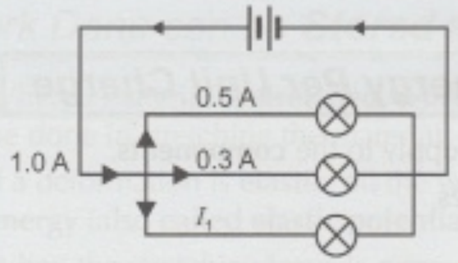


1) As **charge flows** through a circuit, it **doesn't** get **used up** or **lost**.
2) You can easily build a circuit in which the electric current can be **split** between **two wires** — two lamps connected in **parallel** is a good example.
3) Because charge is **conserved** in circuits, whatever charge flows **into** a junction will flow **out** again.
4) Since **current** is **rate of flow of charge**, it follows that whatever **current flows into** a junction is the **same** as the **current flowing out** of it.

   the **sum** of the **currents going into the junction** = the **sum** of the **currents going out**

   This is **Kirchhoff's first law**. It means that the current is the **same** everywhere in a **series circuit**, and is **shared between the branches** of a **parallel circuit**.
5) N.B. — current arrows on circuit diagrams normally show the direction of flow of **conventional current** (see p.25).

EXAMPLE: Use Kirchoff's first law to find the unknown current I_1.



Sum of currents in = sum of currents out

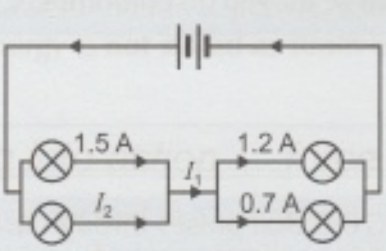
$1.0 = 0.5 + 0.3 + I_1$

$1.0 = 0.8 + I_1$

$I_1 = 1.0 - 0.8$

$I_1 = \mathbf{0.2\ A}$

EXAMPLE: Calculate the missing currents, I_1 and I_2, in this circuit.



Looking at the junction immediately after I_1:

$I_1 = 1.2 + 0.7$

$I_1 = \mathbf{1.9\ A}$

And looking at the junction immediately before I_1:

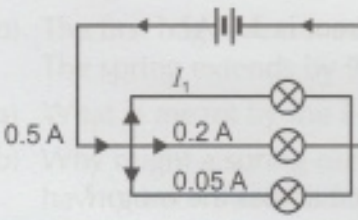
$1.5 + I_2 = 1.9$

$I_2 = 1.9 - 1.5$

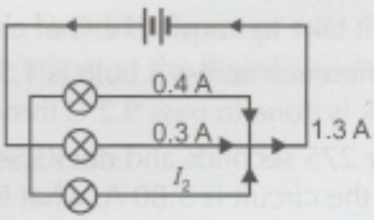
$I_2 = \mathbf{0.4\ A}$

Conserve charge — make nature reserves for circuit boards...

1) What is the value of I_1?



2) What is the value of I_2?



Potential Difference in Electric Circuits

Energy is Always Conserved in Circuits

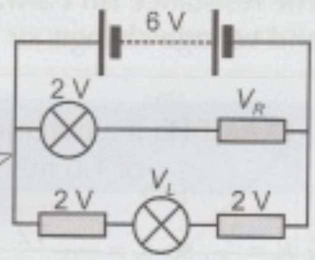
1) Energy is **given** to **charged particles** by the **power supply** and **taken off them** by the **components** in the circuit.
2) Since energy is **conserved**, the **amount** of energy one coulomb of charge loses when going around the circuit must be **equal to** the energy it's **given** by the power supply.
3) This must be true **regardless** of the **route** the charge takes around the circuit. This means that:

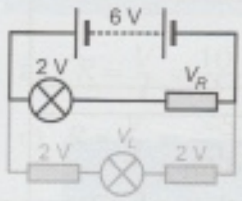
For any **closed loop** in a circuit, the **sum** of the **potential differences** across the components **equals** the **potential difference** of the **power supply**.

This **Kirchhoff's second law.** It means that:

- In a **series circuit**, the potential difference of the power supply is split between all the components.
- In a **parallel circuit**, each **loop** has the same potential difference as the power supply.

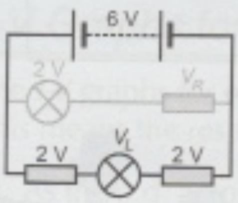
EXAMPLE: Use Kirchoff's second law to calculate the potential differences across the resistor, V_R, and the lamp, V_L, in the circuit shown on the right.





First look at just the top loop:

p.d. of power supply = sum of p.d.s of components in top loop
$6 = 2 + V_R$
So $V_R = 6 - 2 =$ **4 V**

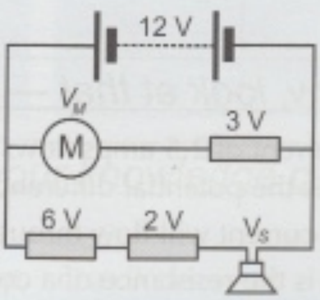


Now look at just the outside loop:

p.d. of power supply = sum of p.d.s of components in outside loop
$6 = 2 + V_L + 2$
So $V_L = 6 - 2 - 2 =$ **2 V**

This page is potentially tricky — so have a read of it all again...

1) For the circuit on the right, calculate:
   a) the voltage across the motor, V_M.
   b) the voltage across the loudspeaker, V_S.
2) A third loop containing two filament lamps is added to the circuit in parallel with the first two loops. What is the sum of the voltages of the two filament lamps?



Resistance

Resistance — The Ratio of Potential Difference to Current

1) If there's a potential difference **across** a component a **current** will **flow through it**.
2) Usually, as the **potential difference** is **increased** the **current increases** — this makes sense if you think of the potential difference as a kind of force **pushing** the charged particles.
3) You can link current and potential difference by defining "**resistance**":

$$\textbf{Resistance of component (in ohms, } \Omega) = \frac{\textbf{potential difference across component} \text{ (in volts, V)}}{\textbf{current passing through component} \text{ (in amps, A)}}$$

Or, in symbols: $R = \frac{V}{I}$

Multiplying both sides by I gives: $V = I \times R$

4) Components with a **low resistance** allow a **large** current to flow through them, while components with a **high resistance** allow only a **small** current.
5) The resistance **isn't** always **constant** though — it can take **different values** as the **current** and **voltage change**, or it can change with conditions like **temperature** and **light level.**

EXAMPLE: If a potential difference of 12 V across a component causes a current of 1.0 mA to flow through it, what is the resistance of the component?

$R = \frac{V}{I}$, so $R = \frac{12}{1.0 \times 10^{-3}} = \mathbf{12\ 000\ \Omega}$, or 12 kΩ

EXAMPLE: What potential difference must be applied across a lamp with a resistance of 200 Ω in order for a current of 0.2 A to flow through it?

$V = I \times R$, so $V = 0.2 \times 200 = \mathbf{40\ V}$

EXAMPLE: What current will flow through an 850 Ω resistor if a potential difference of 6.3 V is applied across it?

$V = I \times R$. Dividing both sides by R gives $I = \frac{V}{R}$,

so $I = \frac{6.3}{850} = 0.007411... = \mathbf{0.0074\ A}$ (or 7.4 mA) **(to 2 s.f.)**

Ohm my, look at that — more questions to do...

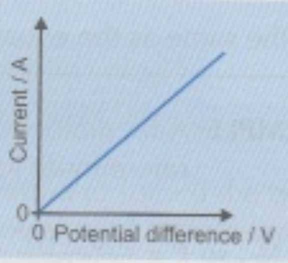
1) If a current of 2.5 amps flows through a component with a resistance of 15 ohms, what is the potential difference across the component?
2) What current will flow through a 2500 Ω resistor if the voltage across it is 6.0 volts?
3) What is the resistance of a component if 1.5 volts drives a current of 0.024 amps through it?

I-V Graphs

Ohm's Law Says Potential Difference is Proportional to Current

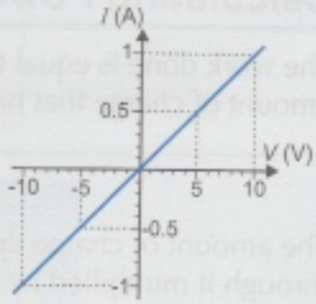
1) An *I-V* graph is a graph of **current** against **potential difference** for a component. For any *I-V* graph, the **resistance** at a given point is the potential difference divided by the current ($R = \frac{V}{I}$).
2) Provided the **temperature** is **constant**, the **current** through an **ohmic component** (e.g. a resistor) is **directly proportional** to the **potential difference** across it ($V \propto I$). This is called **Ohm's Law**.
3) An **ohmic component's** *I-V* graph is a **straight line**, with a gradient equal to 1 ÷ the resistance of the component. The **resistance** (and therefore the **gradient**) is **constant**.

- So for an ohmic component, **doubling** the **potential difference doubles** the **current**.
- Often **external factors**, such as **temperature**, will have a **significant effect** on resistance, so you need to remember that Ohm's law is **only** true for components like resistors at **constant temperature**.



4) Sometimes you'll see a graph with **negative** values for p.d. and current. This just means the current is flowing the **other way** (so the terminals of the power supply have been switched).

EXAMPLE: Look at the *I-V* graph for a resistor on the right. What is its resistance when the potential difference across it is: a) 10 V, b) 5 V, c) –5 V, d) –10 V?



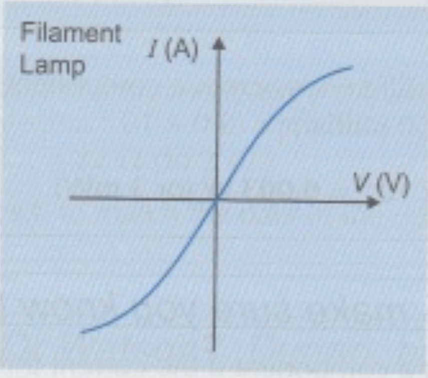
a) $R = \frac{V}{I} = \frac{10}{1} = 10\ \Omega$ b) $R = \frac{V}{I} = \frac{5}{0.5} = 10\ \Omega$

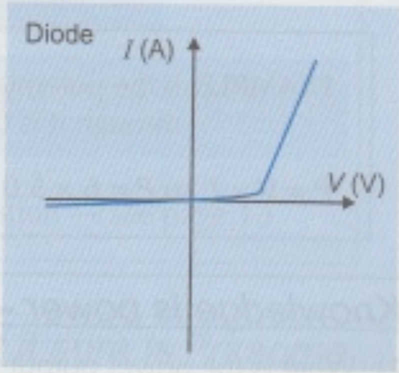
c) $R = \frac{V}{I} = \frac{-5}{-0.5} = 10\ \Omega$ d) $R = \frac{V}{I} = \frac{-10}{-1} = 10\ \Omega$

I-V Graphs for Other Components Aren't Straight Lines

The *I-V* graphs for **other** components **don't** have **constant gradients**. This means the resistance **changes** with voltage.

1) As the p.d. across a filament lamp gets **larger**, the filament gets **hotter** and its resistance **increases**.
2) Diodes only let current flow in **one direction**. The resistance of a diode is **very high** in the **other** direction.





I-Ve decided you need amp-le practice to keep your knowledge current...

1) State Ohm's law.
2) Sketch *I-V* graphs for: a) an ohmic resistor, b) a filament lamp, c) a diode.

Power in Circuits

Power — the Rate of Transfer of Energy

1) Components in electrical circuits transfer the **energy** carried by electrons into other forms.
2) The **work done each second** (or the **energy transferred each second**) is the **power** of a component:

$$\textbf{power}\text{ (in watts, W)} = \frac{\textbf{work done}\text{ (in joules, J)}}{\textbf{time taken}\text{ (in seconds, s)}}$$

Or, in symbols: $P = \frac{W}{t}$

This is the same as the equation for mechanical power that you saw on page 20.

EXAMPLE: A lift motor does 3.0×10^5 J of work in a single one-minute journey. At what power is it working?

$P = \frac{W}{t}$, so $P = \frac{3.0 \times 10^5}{60}$ = **5000 W** (or 5 kW)

Calculating Power from Current and Potential Difference

The work done is equal to the potential difference across the component multiplied by the amount of charge that has flowed through it ($W = V \times Q$) — see p.25.

So: $P = \frac{V \times Q}{t}$

The amount of charge that flows through a component is equal to the current through it multiplied by the time taken ($Q = I \times t$) — see p.25 again.

So: $P = \frac{V \times I \times t}{t}$

Cancelling the t's gives: $P = V \times I$

power (in watts) = **potential difference** (in volts) × **current** (in amps)

EXAMPLE: If the potential difference across a component is 6 volts and the current through it is 0.50 milliamps (5.0×10^{-4} amps), at what rate is it doing work?

$P = V \times I$, so $P = 6 \times 5.0 \times 10^{-4}$ = **0.003 W** (or 3 mW)

Knowledge is power — make sure you know these power equations...

1) What is the power output of a component if the current through it is 0.12 amps when the potential difference across it is 6.5 volts?
2) An electric heater has an operating power of 45 W.
   a) What current passes through the heater when the potential difference across it is 14 volts?
   b) How much work does the heater do in 12 seconds?

Power in Circuits

You Can *Combine* the Equations for *Power and Resistance*

You can **combine** the last equation for the power of an electrical component, $P = V \times I$, with the **equation** for resistance, $R = \frac{V}{I}$ (see p.28), to create two **more useful** equations.

1) Substitute $V = I \times R$ into $P = V \times I$ to get: $P = I \times R \times I = I^2R$

power (in watts) = [**current** (in amps)]^2 × **resistance** (in ohms)

2) Or substitute $I = \frac{V}{R}$ into $P = V \times I$ to get: $P = V \times \frac{V}{R} = \frac{V^2}{R}$

$$\text{power (in watts)} = \frac{[\text{potential difference (in volts)}]^2}{\text{resistance (in ohms)}}$$

Here are some examples — the key here is choosing the **right equation** to use. If the question gives you the value of two variables and asks you to find a third, you should choose the equation that relates these three variables. You might have to **rearrange** it before using it.

EXAMPLE: What is the power output of a component with resistance 100 Ω if the current through it is 0.2 A?

$P = I^2R$, so $P = 0.2^2 \times 100 =$ **4 W**

EXAMPLE: Resistors get hotter when a current flows through them. If you double the current through a resistor, what happens to the amount of heat energy produced every second?

It **increases by a factor of 4** — this is because the current is squared in the expression for the power (you can substitute some values of I and R in to check this).

EXAMPLE: If a lamp has an operating power of 6.5 W and the potential difference across it is 12 V, what is its resistance?

$P = \frac{V^2}{R}$, so multiplying both sides by R gives $P \times R = V^2$, and dividing by P gives:

$R = \frac{V^2}{P}$, so $R = \frac{12^2}{6.5} = 22.153... =$ **22 Ω (to 2 s.f.)**

(This answer is rounded to 2 s.f. to match the data in the question — see page 1.)

Watts up with your watch, Dr Watson? Dunno, but it sure is i²rksome...

1) What is the power output of a 2400 Ω component if the current through it is 1.2 A?
2) A motor has a resistance of 100 Ω. How much work does it do in 1 minute if it is connected to a 6 V power supply?
3) The current through a 6.0 W lamp is 0.50 A. What is the resistance of the lamp?

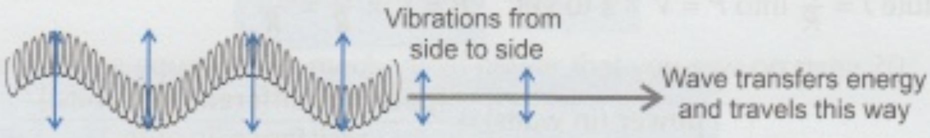
Waves

Waves Transfer Energy Without Transferring Matter

1) Waves are **oscillations** that transfer energy — like water waves or electromagnetic waves.
2) Waves carry **energy** from one place to another **without** transferring **matter**.

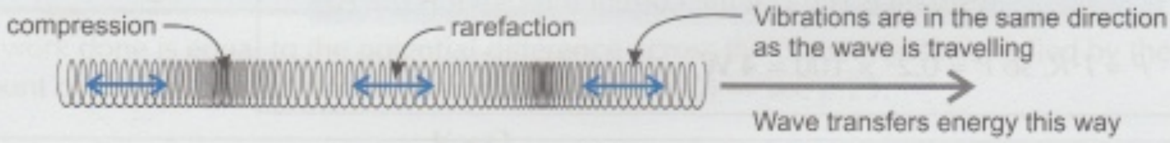
Transverse Waves Vibrate at 90° to the Direction of Travel

Transverse waves have **vibrations** at **90°** to the direction of **energy transfer** and **travel**.
E.g. **electromagnetic** waves (like light) or shaking a Slinky® spring from side to side.



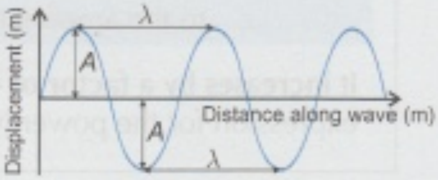
Longitudinal Waves Vibrate Along the Direction of Travel

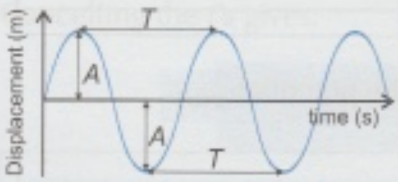
Longitudinal waves vibrate in the **same direction** as the direction of **energy transfer** and **travel**.
They are made of alternate **compressions** and **rarefactions** of the medium.
E.g. sound waves or pushing on the end of a Slinky® spring.



You Can Show Wave Motion on a Graph

A **displacement-distance** graph shows **how far** each part of the wave is **displaced** from its **equilibrium position** for different distances along the wave.





You can also consider **just one point** on a wave and plot how its **displacement** changes with **time**. This is a **displacement-time** graph.

Displacement = how far a point on the wave has moved from its equilibrium position
Amplitude (*A*) = the largest possible displacement from the equilibrium position
Wavelength (λ) = the length of one wave cycle, from crest to crest or trough to trough
Period (*T*) = the time taken for a whole cycle (vibration) to complete, or to pass a given point

Transverse waves are terrible singers — they always skip the chorus...

1) Sketch a graph of displacement against distance for five full wavelengths of a wave with amplitude 0.01 metres and wavelength 0.02 metres.
2) Sketch a graph of displacement against time for three complete oscillations of one part of a wave of amplitude 0.05 metres and time period 0.8 seconds.

Frequency and the Wave Equation

Frequency is the Number of Oscillations per Second

If a wave has a **time period** of 0.2 seconds, it takes 0.2 seconds for a point on the wave to complete **one full oscillation**. So in one second the point will complete **5 full oscillations**. The number of oscillations that one point on a wave completes every second is called the **frequency** of the wave. It has the symbol ***f*** and is measured in **hertz** (Hz).

So a wave with a time period of 0.2 seconds has a **frequency** of 5 hertz.

The equation for frequency is:

$$\textbf{Frequency} = \frac{1}{\textbf{time period}} \quad \text{or} \quad f = \frac{1}{T}$$

EXAMPLE: A wave has a frequency of 350 Hz. What is the period of oscillation of one point on that wave?

$T = \frac{1}{f} = \frac{1}{350} = 0.002857... = \mathbf{0.0029\ s\ (to\ 2\ s.f.)}$

The Wave Equation Relates Speed, Frequency and Wavelength

For a wave of **frequency** ***f*** (in hertz), **wavelength** λ (in metres) and **wave speed** ***v*** (in metres per second) the wave equation is:

$$\textbf{speed = frequency × wavelength} \quad \text{or} \quad v = f \times \lambda$$

EXAMPLE: Sound is a longitudinal wave. If a sound with a frequency of 250 Hz has a wavelength of 1.32 metres in air, what is the speed of sound in air?

$v = f \times \lambda = 250 \times 1.32 = \mathbf{330\ ms^{-1}}$

EXAMPLE: All electromagnetic waves travel at 3.0×10^8 ms^{-1} in a vacuum. If a radio wave has a wavelength of 1.5 km in a vacuum, what is its frequency?

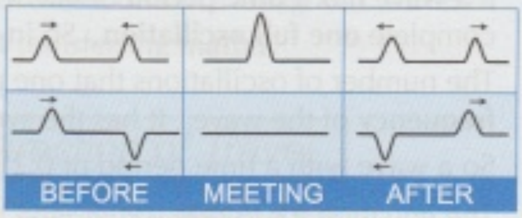
$v = f \times \lambda$, so $f = \frac{v}{\lambda} = \frac{3.0 \times 10^8}{1.5 \times 10^3} = \mathbf{200\ 000\ Hz}$ (or 200 kHz)

Wave equation: lift arm + oscillate hand = pleasant non-vocal greeting...

1) A radio wave has a frequency of 6.25×10^5 Hz.
   What is the time period of the radio wave?
2) A sound wave has a time period of 0.0012 s. Find the frequency of the sound.
3) A wave along a spring has a frequency of 3.5 Hz and a wavelength of 1.4 m.
   What is the speed of the wave?
4) A wave has time period 7.1 s and is moving at speed 180 ms^{-1}.
   a) What is the frequency of the wave?
   b) What is the wavelength of the wave?

Superposition of Waves

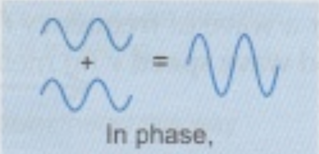
Superposition** Happens When **Two Waves Meet



1) If two waves meet (e.g. waves on a rope travelling in opposite directions), their displacements will briefly **combine**.
2) They become **one single wave**, with a **displacement** equal to the displacement of each individual wave **added together**.
3) This is called **superposition**.
4) If two **crests** meet, the **heights** of the waves are **added together** and the crest height **increases**. This is called **constructive interference** because the **amplitude** of the superposed waves is **larger** than the amplitude of the individual waves.
5) If the **crest** of one wave meets the **trough** of another wave, you **subtract** the trough **depth** from the crest **height**. So if the crest height is **the same** as the trough depth they'll **cancel out**. This is called **destructive interference** because the **amplitude** of the superposed waves is **smaller** than that of the individual waves.
6) After combining, the waves then carry on **as they were** before.

*If Waves are **In Phase** they **Interfere Constructively***

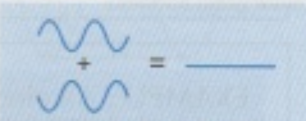
1) Two waves travelling in the **same direction** are **in phase** with each other when the **peaks** of one wave **exactly line up** with the **peaks** of the **other**, and the **troughs** of one wave **exactly line up** with the **troughs** of the **other**.
2) If these waves are **superposed**, they **interfere constructively**. The **combined amplitude** of the final wave is equal to the **sum** of the individual waves.



In phase, constructive interference

*If Waves are **Out of Phase** they **Interfere Destructively***

1) Two waves are **exactly out of phase** if the **peaks** of one wave line up with the **troughs** of the other (and vice versa).
2) If these waves are **superposed**, they **interfere destructively**. If the individual waves had the same amplitude originally, they will **cancel each other out**.



Out of phase, destructive interference

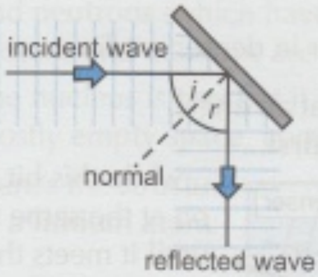
Constructive interference — getting woken up early by noisy builders...

1) What is meant by:
   a) superposition?
   b) constructive interference?
   c) destructive interference?
2) A wave with an amplitude of 0.67 mm is superposed with an identical wave with the same amplitude. The waves are in phase. What is the amplitude of the superposed wave?
3) Two waves, both of amplitude 19.1 m, are exactly out of phase. What is the amplitude of the single wave formed when they superpose?
4) A wave with an amplitude of 35 cm is in phase with a 41 cm amplitude wave. The waves meet and constructive interference occurs. What is the amplitude of the combined wave?

Reflection and Diffraction

Waves can be Reflected

1) When a wave hits a **boundary** between one medium and another, some (or nearly all) of the wave is **reflected back**.

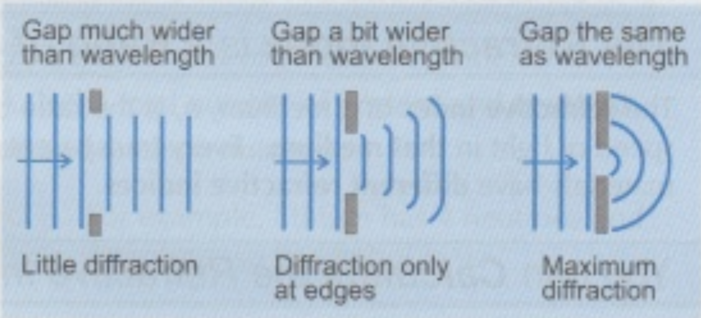


2) The angle of the **incident** (incoming) wave is called the **angle of incidence**, and the angle of the **reflected** wave is called the **angle of reflection.**
3) The angles of incidence and reflection are both **measured from the normal** — an imaginary line running **perpendicular** to the **boundary**.
4) The **law of reflection** says that:

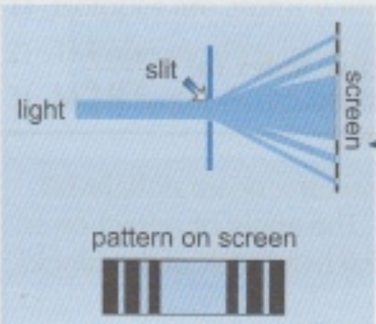
angle of incidence (i) = angle of reflection (r)

Diffraction — Waves Spreading Out

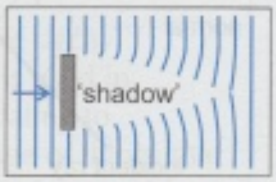
1) Waves **spread out** (**'diffract'**) at the edges when they pass through a **gap** or **pass an object**.
2) The **amount** of diffraction depends on the **size** of the gap relative to the **wavelength** of the wave. The **narrower the gap**, or the **longer the wavelength**, the more the wave spreads out.



3) A **narrow gap** is one about the same size as the **wavelength** of the wave. So whether a gap counts as narrow or not depends on the wave.



4) If light is shone at a **narrow slit** about the **same width** as the **wavelength** of the light, the light **diffracts**.
5) The diffracted light forms a **diffraction pattern** of **bright** and **dark fringes**. This pattern is caused by **constructive** and **destructive interference** of light waves (see p.34).
6) You get diffraction around the edges of **obstacles** too.
7) The **shadow** is where the wave is **blocked.** The **wider** the obstacle compared to the **wavelength**, the **less diffraction** it causes, so the **longer** the shadow.



Mind the gap between the train and the platform — you might diffract...

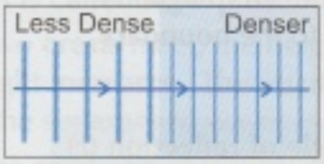
1) What is the law of reflection?
2) Sketch a diagram of a light wave being reflected at an angle by a mirror. Label the incident and reflected waves, the normal, the angle of incidence and the angle of reflection.
3) A water wave travels through a gap about as wide as its wavelength. The gap is made slightly larger. How will the amount of diffraction change?
4) What happens when light is shone at a slit about the same size as its wavelength?

Refraction

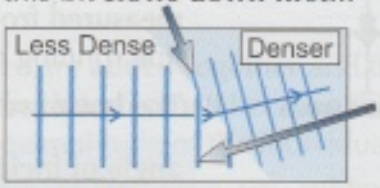
Waves can be Refracted

1) Reflection isn't all that happens when a wave meets a boundary. Usually, some of it is **refracted** too — it passes through the boundary and **changes direction**.
2) Waves travel at **different speeds** in **different media**.
   E.g. — electromagnetic waves, like light, usually travel slower in denser media.

If a wave hits a boundary 'face on', it **slows down** without changing direction.



But if the wave hits at an angle, this bit **slows down first**...



...while this bit carries on at the same speed until it meets the boundary. The wave **changes direction**.

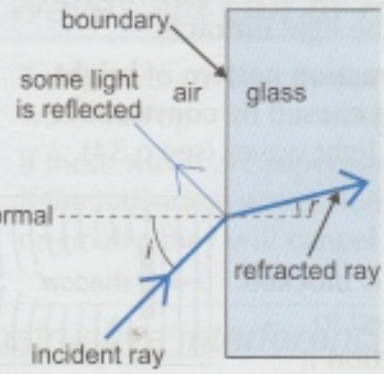
When an electromagnetic wave enters a **denser** medium, it bends **towards** the normal. When one enters a **less dense** medium, it bends **away** from the normal.

The Refractive Index is a Ratio of Speeds

The **refractive index** of a medium, *n*, is the **ratio** of the speed of light in a **vacuum** to the speed of light in **that medium**. **Every** transparent material has a refractive index and different materials have **different refractive indices**.

You can Calculate the Refractive Index using Snell's Law

When an **incident ray** travelling in **air** meets a boundary with **another material**, the **angle of refraction** of the ray, *r*, depends on the **refractive index** of the material and the **angle of incidence**, *i*.



This is called **Snell's Law**. **refractive index (*n*) = $\frac{\sin i}{\sin r}$**

EXAMPLE: The angle of incidence of a beam of light on a glass block is 65°. The angle of refraction is 35°. What is the refractive index of the block?

$$n = \frac{\sin i}{\sin r} = \frac{\sin 65}{\sin 35} = 1.580... = \mathbf{1.6}$$

You can **rearrange** Snell's Law to find an angle of refraction or incidence, e.g. $r = \sin^{-1}(\frac{\sin i}{n})$.

This page has a high refractive index — it's really slowed me down...

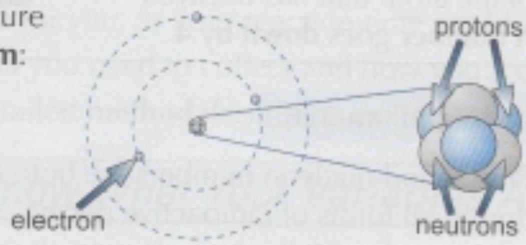
1) A wave hits a boundary between two media head on. Describe what happens to the wave.
2) A wave hits a boundary between two media at an angle. Describe what happens to the wave.
3) A light wave travelling in air hits a transparent material at an angle of 72° to the normal to the boundary. The angle of refraction is 39°. What is the refractive index of the material?
4) A light wave hits the surface of the water in a pond at 23° to the normal. The refractive index of the pond water is 1.3. What is the angle of refraction?

Atomic Structure

*Atoms are **Made Up** of **Three** Types of **Particle***

1) According to the **nuclear model**, the atom is made up of electrons, protons and neutrons.
2) The **nucleus** is at the **centre** of the atom. It contains **protons** (which have a **positive** charge) and **neutrons** (which have **no charge**), giving the nucleus an **overall positive charge.** Protons and neutrons are both known as **nucleons.**
3) The nucleus is **tiny** but it makes up **most** of the **mass** of the atom. The rest of the atom is mostly **empty space**, containing only the negative **electrons** which orbit **around** the nucleus.

Here's the structure of a **lithium atom**:



	relative mass	relative charge
proton	1	+1
neutron	1	0
electron	0.0005	−1

Atomic Structure can be Represented Using Nuclide Notation

1) The **proton number** (or atomic number), **Z**, is the number of **protons** in an atom.
2) The **nucleon number** (or mass number), **A**, is the total number of **protons** and **neutrons.**
3) An element can be **described** by its **proton** and **nucleon numbers:**

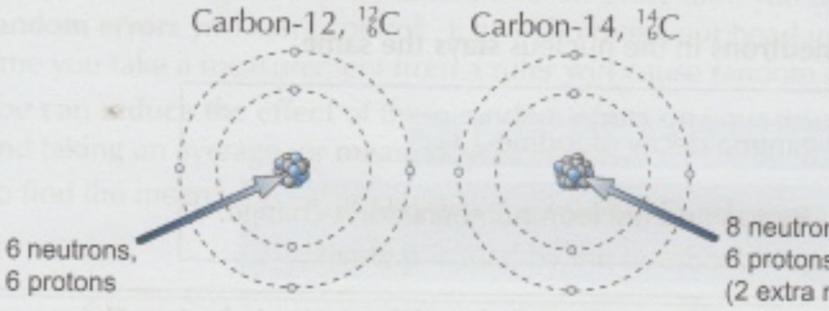
nucleon number → $^{A}_{Z}X$ ← element symbol
proton number →

For example, lithium has 4 neutrons and 3 protons, so its symbol is $^{7}_{3}Li$.

Isotopes are Different Forms of the Same Element

1) Isotopes are atoms with the **same number** of **protons** but a **different number** of **neutrons.**
2) This means they have the **same proton number**, but **different nucleon numbers.**
3) Many isotopes are **unstable** and give off **radiation** (see next page).

EXAMPLE: Carbon-12 and carbon-14 are two isotopes of carbon.



The radioactive decay of carbon-14 is used in radiocarbon dating to estimate the age of things that are thousands of years old.

Radiocarbon dating — what physicists do on Valentine's Day...

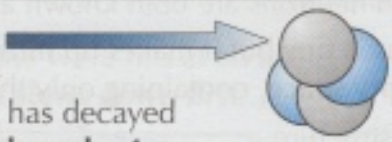
1) How many protons and neutrons are there in each of the following nuclei?
a) $^{241}_{95}Am$ b) $^{239}_{94}Pu$ c) $^{90}_{38}Sr$ d) $^{60}_{27}Co$ e) $^{226}_{88}Ra$
2) What is an isotope of an element?

Nuclear Radiation

If an atom is **unstable**, it can undergo **radioactive decay** and give off **nuclear radiation**. By decaying, a nucleus emits **particles** or **energy**, making it **more stable**.

There are **three** kinds of nuclear radiation you need to know about:

*In **Alpha Decay** (Symbol α), an Alpha Particle is Emitted*



1) An **alpha particle** is emitted from the **nucleus**. It is made up of **two protons** and **two neutrons**.
2) As a result, the **proton number** of the atom that has decayed goes **down by 2** and the **nucleon number** goes **down by 4**.

EXAMPLE: The alpha decay of radium-226.

$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}\alpha$ Proton and nucleon numbers are both conserved during all forms of radioactive decay:

Nucleon number: $226 = 222 + 4$ Proton number: $88 = 86 + 2$

*In **Beta Decay** (Symbol β), an **Electron** is Emitted*

1) A **neutron** in the nucleus turns into a **proton** and an **electron**. The electron is **emitted** from the nucleus and is called a **beta particle**.
2) As a result the **proton number** of the nucleus goes **up by 1**, but the **nucleon number doesn't change**.

EXAMPLE: The beta decay of radium-228.

$^{228}_{88}Ra \rightarrow ^{228}_{89}Ac + ^{0}_{-1}\beta$ Nucleon number: $228 = 228 + 0$
Proton number: $88 = 89 - 1$

***Gamma Decay** (Symbol γ) Emits **Electromagnetic** Radiation*

1) High-energy **electromagnetic radiation**, called **gamma radiation** is **emitted** from the nucleus.
2) The **number** of **protons** and **neutrons** in the nucleus **stays the same**.

EXAMPLE: The gamma decay of iodine-131.

$^{131}_{53}I \rightarrow ^{131}_{53}I + ^{0}_{0}\gamma$ Proton and nucleon numbers don't change.

You beta learn this radiation stuff — I promise it's not alpha nothing...

1) What is an alpha particle made up of?
2) Describe what happens during the emission of beta and gamma radiation.
3) Complete the following decay equations by filling in any missing radiation symbols, proton numbers or nucleon numbers:
a) $^{242}_{94}Pu \rightarrow ^{\_}_{\_}U + ^{4}_{2}\alpha$ b) $^{\_}_{\_}K \rightarrow ^{40}_{20}Ca + ^{0}_{-1}\beta$ c) $^{222}_{86}Rn \rightarrow ^{218}_{84}Po + ^{\_}_{\_}\_$ d) $^{14}_{6}C \rightarrow ^{\_}_{\_}N + ^{0}_{-1}\beta$

Planning an Experiment and Collecting Data

*Scientists do **Experiments** to **Answer Questions***

You need to **plan experiments** carefully to make sure you get the **best results** possible:

1) Make a **prediction** or **hypothesis** — a testable statement about what you think will happen.
2) Identify your **variables** (see below).
3) Think about any **risks**, and how you can minimise them.
4) Select the right **equipment** for the job — if you're measuring a time interval in minutes you might use a **stopwatch**, but if it's in milliseconds you may need to get a **computer** to measure the time for you, as your reaction time could interfere with your results.
5) Decide what **data** you need to collect and how you'll do it.
6) Write a clear, detailed **method** describing exactly what you're going to do.

*You Need to Know What Your **Variables** Are*

A variable is anything that has the **potential to change** in an experiment.

The **independent variable** is the thing **you change** in an experiment.

The **dependent variable** is the thing you **measure** in an experiment.

All the **other variables** must be kept the **same** to make it a **fair test**. These are **control variables**.

EXAMPLE: An experiment investigates how the height an object is dropped from affects the time it takes to fall. Identify the variables in this experiment.

The **independent variable** is the **height** you drop the object from — it's what you change.
The **dependent variable** is the **time** the object takes to fall — it's what you measure.
Everything else in the experiment should be **controlled**, so no other variables change.
For example, the **same object** should be used throughout the experiment (so its size and mass don't change), the **conditions** in the room you do the experiment in should be constant, and you shouldn't change your measuring **equipment** halfway through.

Repeating** an Experiment Lets You Calculate a **Mean

Normally, you'll get a slightly different result every time you do an experiment, due to small **random errors** you can't control. E.g. — holding your head in a slightly different place each time you take a measurement from a ruler will cause random errors in the length you read off.

You can **reduce the effect** of these random errors on your results by **repeating** your experiment and taking an average, or **mean**, of your results.

To find the mean:

1) **Add together** the **results** of each repeat.
2) **Divide** this total by the number of **repeats** you did.

Independent variables — not keen on accepting help...

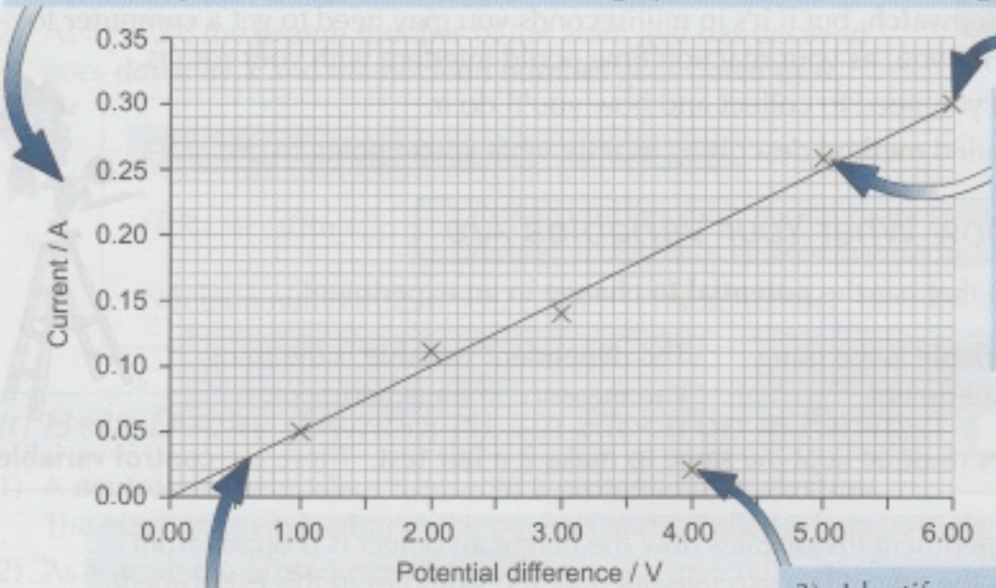
1) A scientist investigates how changing the potential difference across a circuit component affects the current through it. He measures the current three times at each potential difference.
   a) Identify the independent and dependent variables in this investigation.
   b) For a potential difference of 4 V, the scientist records currents of 0.13 A, 0.17 A and 0.12 A. Calculate the mean current through the component when the potential difference is 4 V.

Analysing Your Data

You can Present Your Results on a **Graph**

Graphs are the easiest way to see any **patterns** or **trends** in your results.

1) Usually the **independent variable** goes on the *x*-**axis** (along the bottom) and the **dependent variable** goes on the *y*-**axis** (up the side). Make sure you **label** both axes **clearly** with the quantity and **units**. Pick a **sensible scale** — both axes should go up in sensible steps, and should spread the data out over the full graph (rather than bunching it up in a corner).



2) Plot your points using a **sharp pencil**. This will help make sure they're as **accurate** as possible.

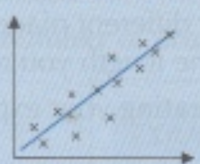
3) Identify any **anomalous results**, like this one — it's way off the general trend, and looks like it was caused by a mistake. **Ignore** anomalous results when drawing your **line of best fit**.

4) Draw a **line of best fit** for your results. Around half the data points should be above your line of best fit and half below it. The line could be **straight** or **curved**, depending on your data.

Graphs Can Show Different Kinds of **Correlation**

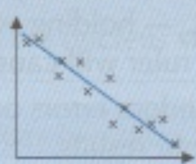
The **correlation** describes the relationship between the variables. Data can show:

POSITIVE CORRELATION:



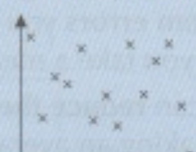
As one variable increases, the other increases.

NEGATIVE CORRELATION:



As one variable increases, the other decreases.

NO CORRELATION:



No relationship between the variables.

Remember, just because two variables are correlated it doesn't mean a change in one is **causing** a change in the other — there could be a third variable affecting them both.

Anomalouse results — unusual results in the insect breeding program...

1) The table on the right gives the speed of a cyclist as he accelerates from rest. Plot a graph of his speed against time, and draw a line of best fit.

time / s	0.0	2.0	4.0	6.0	8.0	10.0
speed / ms^{-1}	0.0	0.7	1.8	2.6	3.2	4.2

Conclusions and Uncertainty

*Draw **Conclusions** that Your Results **Support***

You should draw a conclusion that **explains** what your data shows.

1) Your conclusion should be limited to what you've **actually done** and found out in your experiment. For example, if you've been investigating how the force applied to a spring affects how much it stretches, and have only used forces between 0 and 5 N, you can't claim to know what would happen if you used a force of 10 N, or if you used a different spring.
2) You also need to think about how much you can **believe** your conclusion, by evaluating the **quality** of your results (see below). If you can't trust your results, you can't form a **strong conclusion**.

*You can Never Measure Anything **Exactly***

1) There will always be **errors** and **uncertainties** in your results caused by lots of different things, including **human error** (e.g. your reaction time). The more errors there are in your results, the **lower the quality** of your data. This will affect the strength of your **conclusion** (see above).
2) All measurements will have some uncertainty due to the equipment used. For example, if you measure a length with a ruler, you can only measure it to the nearest millimetre, as that's the **smallest difference** marked on the ruler's scale. If you measure a length with a ruler as 14 mm you can write this as **14 ± 0.5 mm** to show that you could be up to half a millimetre out either way.
3) If you have a value without a ± sign, the number of **significant figures** gives you an estimate of the **uncertainty**. For example, 72 ms^{-1} has **2 significant figures**, so without any other information you know this value must be 72 **± 0.5 ms^{-1}** — if the value was less than 71.5 ms^{-1} it would have been rounded to 71 ms^{-1}, if it was greater than 72.5 ms^{-1} it would have been rounded to 73 ms^{-1}.

*Think About How to **Improve** Your Experiment*

You should always think about how your experiment could be **improved**:

1) Did the experiment actually **test** what it was supposed to? Could you make it more **relevant** to the question?
2) Was there anything you could have done to prevent some of the **errors** in your results?
3) Would different **apparatus** or a different **method** have given you **better results**?

In conclusion, I need a cup of tea...

1) A student records how long it takes for a car to stop when the brakes are fully applied. He uses a stopwatch, and gets a measurement of 7.628 ± 0.0005 seconds.
   a) What is the smallest difference the stopwatch can measure?
   b) The student says from his result he can accurately report the time taken for the car to stop to 4 significant figures. Is he correct? Explain your answer.